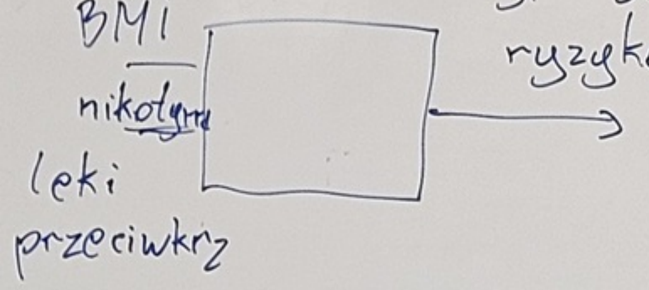
$K \times N$ $\quad 1 \times N$ $\quad 1 \times N$ $\quad K \times 1$

$X \quad Y \quad \hat{Y} \quad \Theta$

$[y_1 \dots y_N] \quad [\hat{y}_1, \dots, \hat{y}_N]$



$$\hat{Y} = \Theta^T X \qquad Y - \hat{Y}$$

$$Q(\Theta) = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 = (Y - \hat{Y})(Y - \hat{Y})^T$$

$$= \sum_{i=1}^{N} (y_i - \Theta^T x_i)^2$$

$y_1 = a x_1 \qquad a = \frac{y_1}{x_1}$ 5s

$\begin{cases} y_1 = a x_1 + b \\ y_2 = a x_2 + b \end{cases}$ 30s

$\begin{cases} y_1 = a x_1^{(1)} + a x \end{cases}$

$$Q = (Y - \Theta^T X)(Y - \Theta^T X)^T =$$

$$= (Y - \Theta^T X)(Y^T - X^T \Theta) =$$

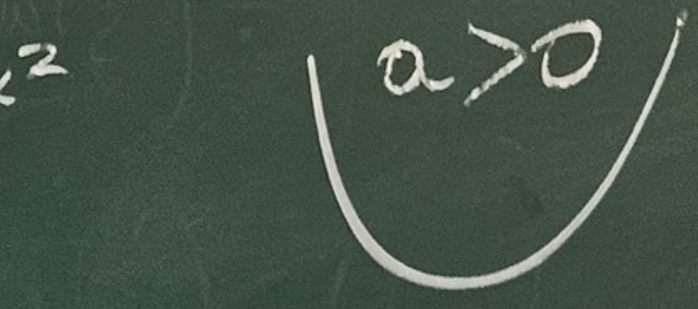
$$= \underbrace{Y Y^T}_{I} - \underbrace{Y X^T \Theta}_{II} - \underbrace{\Theta^T X Y^T}_{III} + \underbrace{\Theta^T X X^T \Theta}_{IV}$$

$$\frac{\partial Q}{\partial a_1}$$

$$\textcircled{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_K \end{bmatrix}$$

$\nabla_{\textcircled{a}} Q$

ax^2



I $\nabla_{\textcircled{\sim}} YY^T = O_K$

II $\nabla_{\textcircled{\sim}} YX^T \textcircled{\sim} = (YX^T)^T = XY^T$

III $\nabla_{\textcircled{\sim}} \textcircled{\sim}^T XY^T = XY^T$

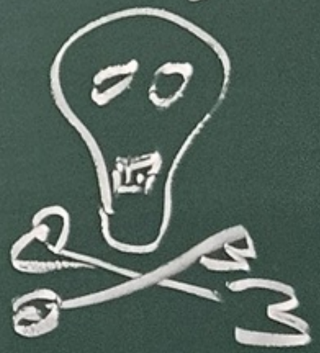
IV $\nabla_{\textcircled{\sim}} \textcircled{\sim}^T XX^T \textcircled{\sim} = 2XX^T \textcircled{\sim}$

a=0



$$\nabla_\Theta Q = I - II - III + IV$$

co dalej?



$$\Theta = (XX^T)^{-1} XY^T$$

$$\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} \end{bmatrix}$$

co to znaczy dla lekarki, że dane pacjentek mają $\det(XX^T) \approx 0$