Computer Science

Jerzy Świątek

Systems Modelling and Analysis

L.2a. Model building task based on experiment - identification problem

Model in the systems research
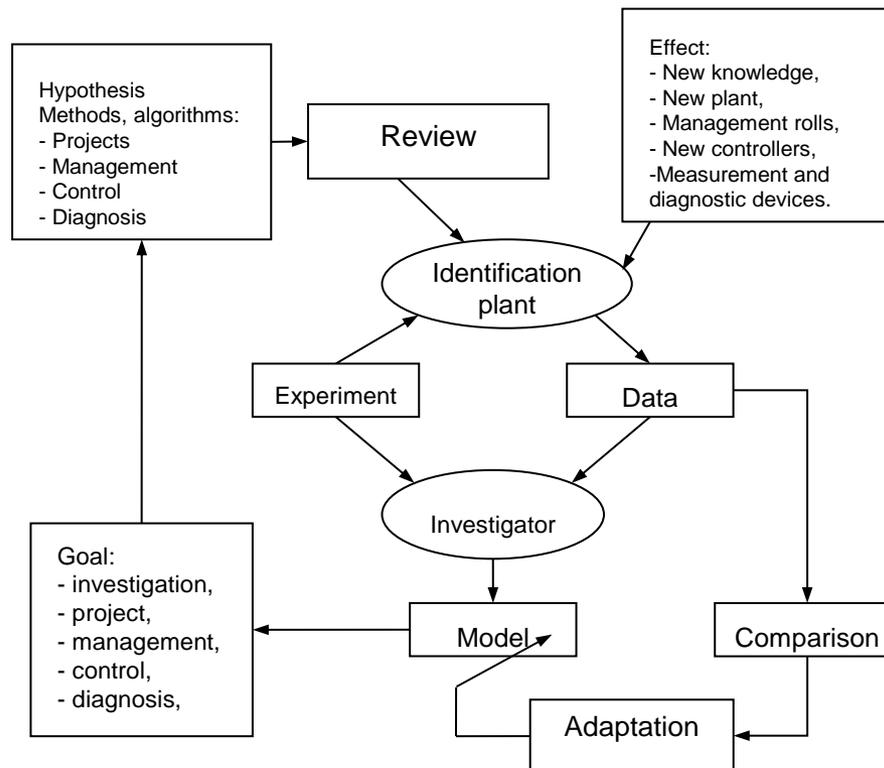
Identification task (1)

Identification task (2)

1. Determination of the identification plant
2. Determination of the class model
3. Experiment organization
4. Determination of the identification algorithms
5. Identifiers realization

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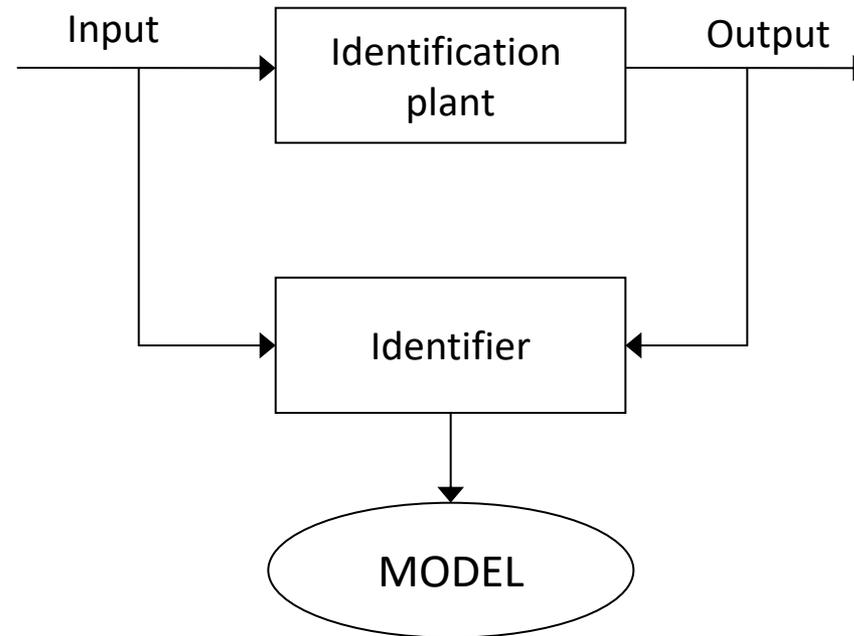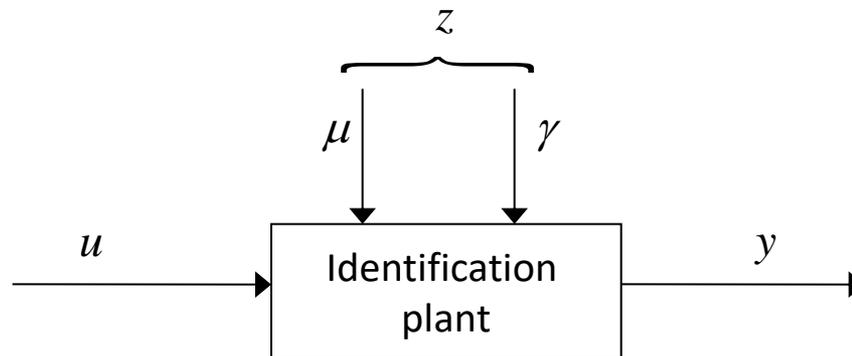
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Ad.1. Determination of the identification plant

u – input
y – output
μ – measured disturbances
γ – unmeasured disturbances

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Ad.2. Determination of the class of model (1)

- Process analysis

- Data analysis

- Arbitrary model

- Expert model
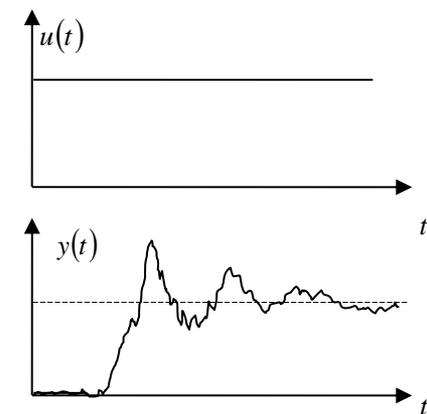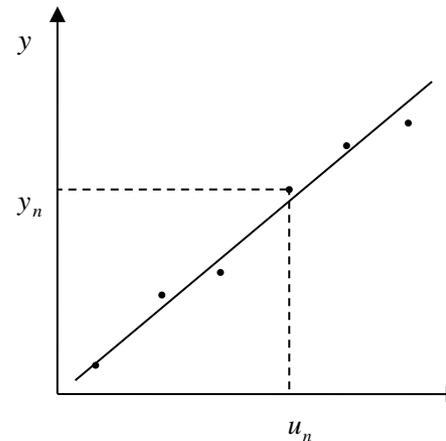
Ad.2. Determination of the class of model (2)

• Plant in the class of model
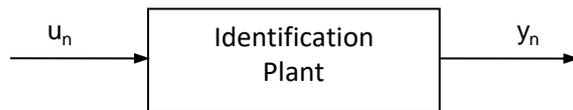
Ad.2. Determination of the class of model (3)

- Choice of the best model
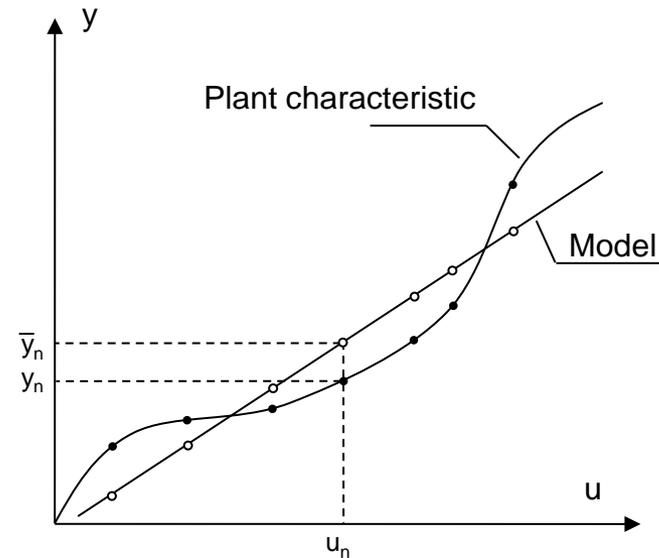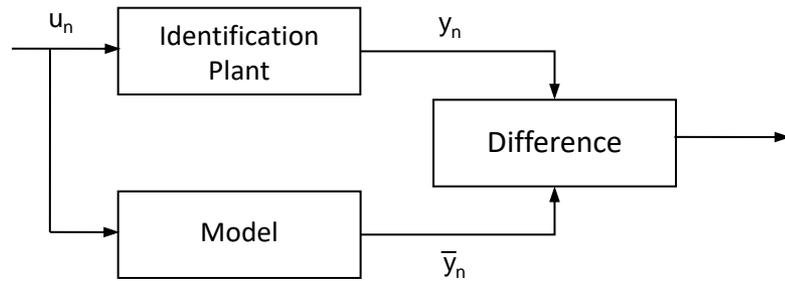
Ad.3. Experiment organization

Static plant
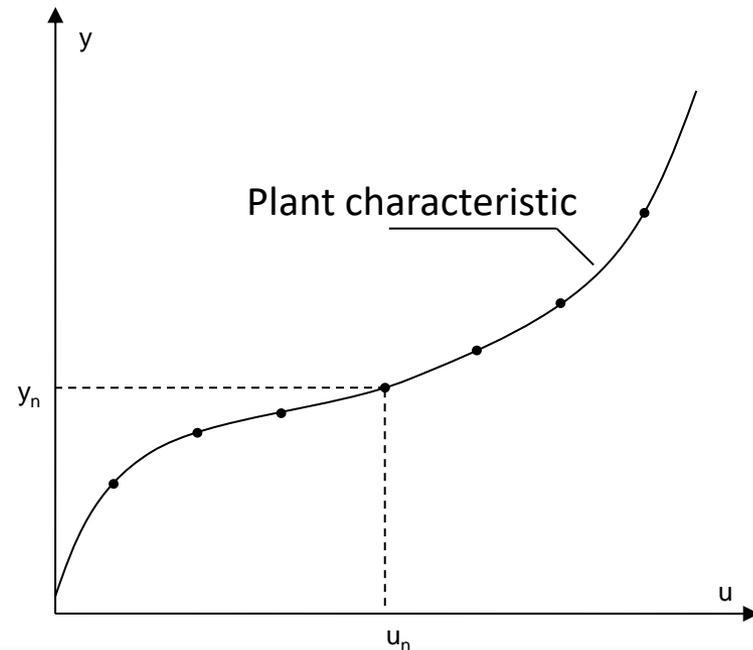$$U_N = \begin{bmatrix} u_1 & u_2 & \cdots & u_N \end{bmatrix}, \quad Y_N = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix}$$

Dynamic plant
$$U_T = \left\{ u(t) \right\}_{t=t_0}^{T}, \quad Y_T = \left\{ y(t) \right\}_{t=t_0}^{T},$$

Discrete type observations
$$t_1, t_2, \cdots, t_N, t_n \in [t_0 \ T], n = 1, 2, \ldots, N$$
$$U_N = \left\{ u(t_n) \right\}_{n=1}^{N}, \quad Y_N = \left\{ y(t_n) \right\}_{n=1}^{N}.$$

Dynamic, discrete type plant
$$U_N = \left\{ u_n \right\}_{n=1}^{N}, \quad Y_N = \left\{ y_n \right\}_{n=1}^{N}.$$

Ad.4. Identification algorithm (1)
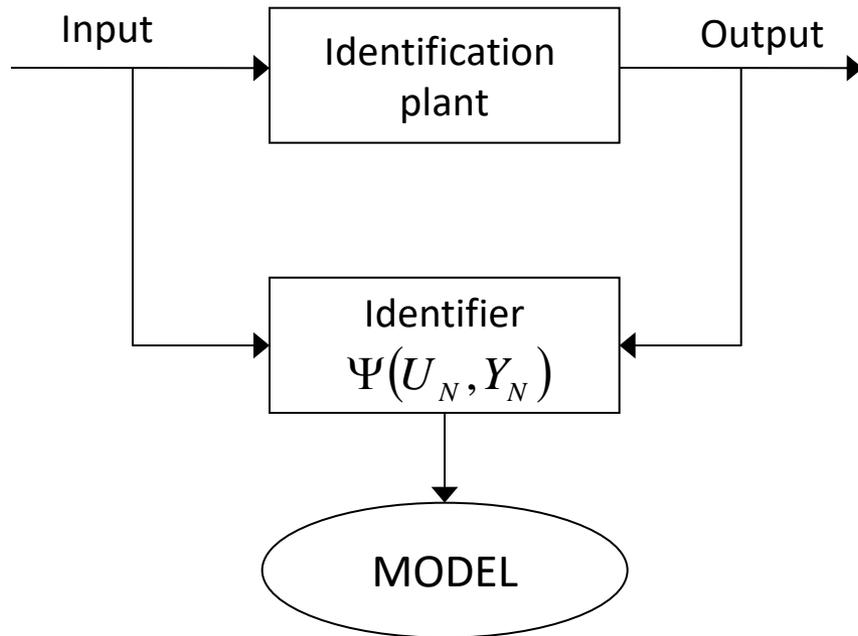
$U_N -$ measurements of input signals

$Y_N -$ measurements of output signals

$\Psi -$ identification algorithm

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Ad.4. Identification algorithm (2)

$$a_N^* = \Psi(U_N, Y_N)$$

$$Q_N \sum_{n=1}^{N} (y_n - au_n)^2$$

$$a_N^* = \frac{\sum_{n=1}^{N} y_n u_n}{\sum u_n^2}$$

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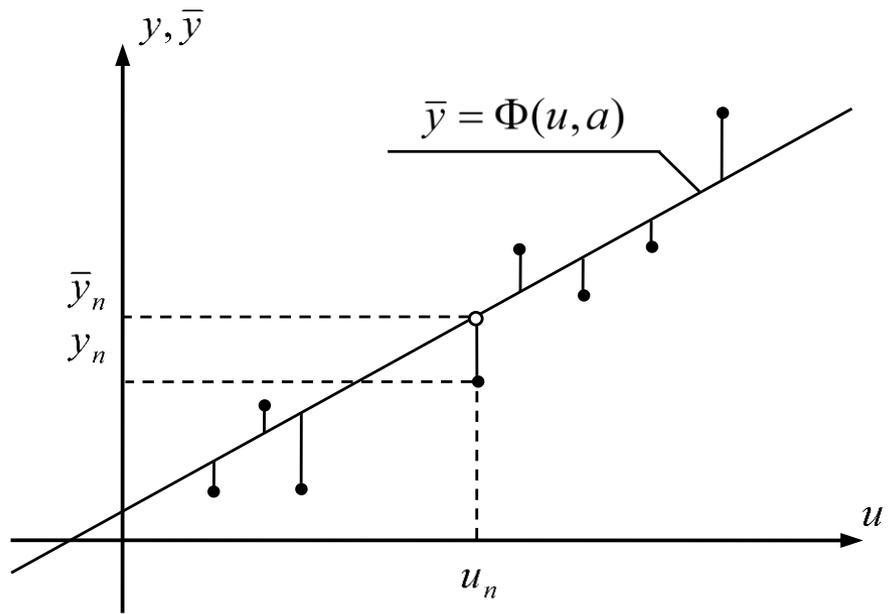
Identifiers realization

- Identification algorithm
  - Computer program
  - Hardware realization

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Plant in the class of model

Deterministyczny

$u_n \rightarrow \boxed{F(u,\theta)} \rightarrow y_n$

$$U_N = \begin{bmatrix} u_1 & u_2 & \cdots & u_N \end{bmatrix},$$
$$Y_N = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix}$$

$$y_n = F(u_n,\theta),$$
$$n = 1, 2, \ldots, N$$

$$Y_N = \overline{F}(U_N,\theta)$$

$$\theta = \overline{F}^{-1}(U_N,Y_N) \overset{df}{=} \Psi_N(U_N,Y_N)$$

$y = F(u,\theta)$

Losowy

$u_n \rightarrow \boxed{F(u,\theta,\omega)} \xrightarrow{y_n} \boxed{h(y,z)} \rightarrow w_n$

with ω_n and z_n inputs

$$\theta_N = \Psi_N(U_N,W_N)$$

$y = F(u,\theta,\omega), \; \omega = \omega_k$
$y = F(u,\theta,\omega), \; \omega = \omega_n$
$y = F(u,\theta,\omega), \; \omega = \omega_m$
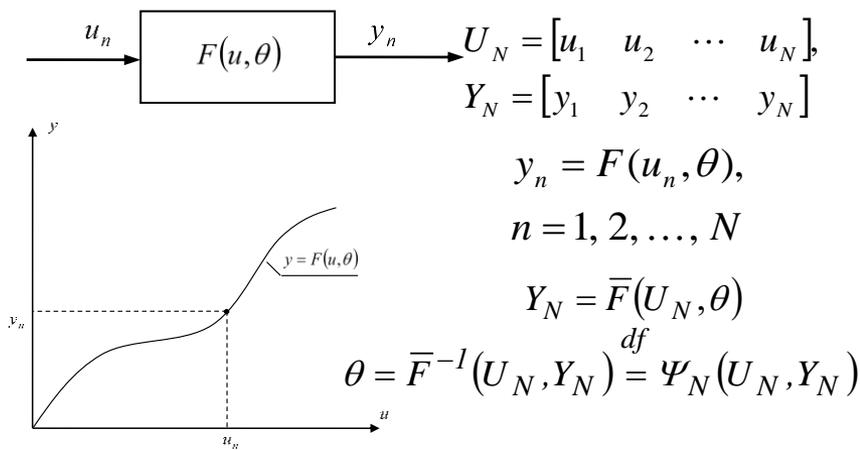$y = F(u,\theta,\omega), \; \omega = \omega_n$

Methods:
- Least square
- Maximum likelihood
- Bayes'a

Choice of the best model

$u_n \rightarrow \boxed{\begin{array}{c}\text{Obiekt}\\\text{identyfikacji}\end{array}} \xrightarrow{y_n}$

$\boxed{\Phi(u,\theta)} \xrightarrow{\overline{y}_n}$

$\boxed{Q_N(\theta)} \xrightarrow{\theta_N^*}$

$$\overline{y} = \Phi(u,\theta_N^*)$$
$$\theta_N^* \rightarrow Q_N(\theta_N^*) = \min_\theta Q_N(\theta)$$
$$\theta_N^* = \Psi_N(U_N,Y_N)$$
$$U_N = \begin{bmatrix} u_1 & u_2 & \cdots & u_N \end{bmatrix},$$
$$Y_N = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix}$$

Optimal model for:
- Given measurements sequences
- Klass of models
- Given performance index

$y = F(u)$
$\overline{y} = \Phi(u,\theta)$

$$\overline{y} = \Phi^*(u) = \mathrm{E}[y|u=u]$$
$$\overline{y} = \Phi(u,\theta^*)$$
$\mathrm{E}[y|u=u']$
$f_y(y|u')$
$f_y(y|u)$
$f_u(u)$

Full probabilistic knowledge
- First type regrsesion
- Second type regression

Unknown probabilostic knowledge
- Performance index estimatin
- Parameter estimation of the probability distribution
- Probabilit ditribution estimatoin

Thank you for attention

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